Recovery Rates, Default Probabilities, and the Credit Cycle

Max Bruche and Carlos González-Aguado

CEMFI
Motivation

Recovery Rates, Default Probabilities, and the Credit Cycle

Max Bruche and Carlos González-Aguado
The research questions & method

• Can you exploit the joint time-series behaviour of these variables to look at credit risk?

⇒ Let both default probabilities and recovery rate distributions be driven by an unobserved factor (Markov chain).

• How bad is it to treat recovery rates as constant?
Related literature

• Default probabilities vary with the cycle. (Bangia et al. (2002), Nickell et al. (2000))

• Recovery rates $\uparrow \implies$ default probabilities $\downarrow$ (Altman et al. 2006, Acharya et al. 2007)

Is the amplification effect of recovery rates large?

• Recovery rates and default probabilities can be modelled as functions of observed covariates. (Chava et al. 2008)

• Theory:
  • Recovery rates should be related to the state of the industry: Shleifer and Vishny (1992).
  • RBC and credit: Bernanke and Gertler (1989), Williamson (1987)
Summary of results

• Credit variables are more tightly related to each other contemporaneously and over time than to macro variables.

• “Credit cycle” ≠ business cycle.
  ⇒ latent factor approach works well, and better than many models based on observed covariates.

What happens if you allow for time-varying recovery rate distributions? You get
• slightly higher estimates of tail risk, (the 99% VaR e.g. 3.3% → 3.4%, or 3.4% → 3.7%).
• and practically the same expected losses.

⇒ The behaviour of recovery rates has a amplifying effect, but smaller than previously suggested.

Caveats?

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Caveats?
The basic idea

The DGP works as follows:

- The state of the credit cycle is determined by a two-state Markov chain.
- The number of defaulting firms is drawn using the state-dependent default probability.
- For each defaulting firm, we draw a recovery rate from the state-dependent recovery rate distribution.
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Dependence:

- Conditional on the state, defaults are independent, recoveries between firms are independent, and the number of defaulting firms and recoveries are independent.
- As a consequence, (unconditional) dependence is driven entirely by the (unobserved) state of the credit cycle.
Specific assumptions about functional forms
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- Conditional on the state of the cycle, default arrival is described by discrete hazards of the form

\[ \lambda_t = \left(1 + \exp \{\gamma_0 + \gamma_1 c_t + \gamma_2 X_t\}\right)^{-1}. \]

\( (t: \text{time, } c_t: \text{cycle, } X_t: \text{economy-wide variables}) \)
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• Recoveries for each default event are drawn from a beta distribution.

• The parameters of this beta distribution are given by:

\[ \alpha_{ti} = \exp \{ \delta_0 + \delta_1 c_t + \cdots + \delta_6 X_t \} \]  
\[ \beta_{ti} = \exp \{ \zeta_0 + \zeta_1 c_t + \cdots + \zeta_6 X_t \} \]  

\((t: \text{time, } i: \text{firm})\)
Estimation

- The model can be easily be estimated using a version of the Hamilton filter (MLE).
- For this we need the number of defaulting firms, and non-defaulting firms in each period, and a recovery rate for each default event.
Data sources

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- the Altman-NYU Salomon Center Corporate Bond Default Master Database, consisting of prices just after default of more than 2,000 bonds of US firms from 1974 to 2005, with issuers and dates, and a “bond category”,

- Moody’s annual bond issuer default rates. Assuming that both the Altman data and Moody’s data track the same set of firms, we can obtain the number of non-defaulting firms in each year by dividing Altman’s number of defaulting firms by the Moody’s default rate.

- We augment this with GDP growth, investment growth, unemployment, the S&P 500 index, the VIX, the slope of the term structure, and an NBER recession indicator.
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## Macro variables versus the business cycle

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→ Macro variables are significant, but do not contribute much to the fit.
Is the business cycle = credit cycle?

- The estimated credit downturns start earlier than NBER recessions, and end later.
- We investigate lead-lag relationships between macro variables and credit variables and find that recovery rates Granger cause log GDP growth (very significant!).
VaR Simulation (variation in RR and PD)
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- Suppose
  \[ PD|E = 2\% \quad L|E = 30\% \]
  \[ PD|R = 10\% \quad L|R = 70\% \]
  \[ P(E) = 1/2 \]

\[ E[L \cdot PD] = 0.5 \times 30\% \times 0.02 + 0.5 \times 70\% \times 0.1 = 3.8\% \]

\[ E[L] \cdot E[PD] = 3\% = \Rightarrow E[L \cdot PD] - E[L] \cdot E[PD] = 80 \text{bp} \]

i.e. expected loss can be bigger if PD and L covary.

- We could calculate the increase in expected loss from our model, but note that
  \[ E[L \cdot PD] - E[L] \cdot E[PD] = \text{Cov}(L,PD) \]

- In our data, \[ \text{Cov}(\bar{L},dfr) = 6 \text{ bp} \]
**Expected Loss**

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  \[ PD|E = 2\% \quad L|E = 30\% \]
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  \[ P(E) = 1/2 \implies E[L] = 50\%, \, E[PD] = 6\%. \]
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Therefore

\[ E[L \cdot PD] = .5 \times 30\% \times 0.02 + .5 \times 70\% \times 0.1 = 3.8\% \]
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i.e. expected loss can be bigger if \( PD \) and \( L \) covary.

- We could calculate the increase in expected loss from our model, but note that
  \[ E[L \cdot PD] - E[L]E[PD] = \text{Cov}(L, PD) \]

- In our data, \( \text{Cov}(\bar{L}, df) = 6bp \).
Some conclusions

• We propose an econometric model in which default rates and recovery rates are driven by an unobserved Markov chain.

• This describes the data well, and does better than many models based on observed covariates.

• In particular, macro variables are significant, but don’t help much in matching variation in credit risk.

We can use the estimated model to look at what happens when we go from constant to time-varying recovery rate distributions. We get

• slightly higher estimates of tail risk,

• but practically the same expected losses.