Online Appendix for "Debt Maturity and the Liquidity of Secondary Debt Markets."

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In this Online Appendix, we consider two modifications to our baseline model.

First, in Section A, we describe how the model can be extended to incorporate a new class of agents, *marketmakers*, who intermediate between buyers and sellers in the secondary market. We show that bid-ask spreads of marketmakers should be decreasing in refinancing frequencies (i.e. increasing in expected maturities). We provide a numerical example which illustrates that marketmakers can speed up matching substantially, even while the externality is still sizeable.

Second, in Section B, we show that our externality is also present when using the increasing-returns-to-scale matching function more commonly used in the finance literature (see e.g. Duffie, Garleanu, and Pedersen, 2005), rather than our constant-returns-to-scale matching function. We provide a numerical example which illustrates that the thick market externalities inherent in the increasing-returns-to-scale matching function amplify our externality.

In both sections, we simplify our exposition by explicitly only looking for steady-state equilibria that are symmetric in the sense that all firms choose the same refinancing frequency and face value of debt. (As in the baseline model, it could be shown that all steady-state equilibria are in fact symmetric, and no asymmetric equilibria exist.)

All proofs are in Section C.

A Extended model: Marketmakers

To introduce marketmakers, we follow the approach of Duffie, Garleanu, and Pedersen (2005). As in their model, marketmakers are risk-neutral and utility-maximizing agents, who have no funds. Marketmakers can intermediate as follows: They search in the secondary market to be matched with buyers and sellers, via a special matching technology (described below), and also have access to an inter-dealer market in which they can instantly unload the positions which they enter into with investors in the secondary market (so that they hold no inventory at any time). Marketmakers set bid (B) and ask (A) prices at which they are willing to buy from and sell to investors, respectively, which are determined through Nash bargaining. Marketmakers do not need funds to intermediate because they can instantly off-set any position taken with a final investor in the inter-dealer market at a price Q that in equilibrium satisfies $B \leq Q \leq A$.

We depart from the assumptions in Duffie, Garleanu, and Pedersen (2005) in two key points: We assume a constant-returns-to-scale matching function for matches between investors and marketmakers, as opposed to an increasing-returns-to-scale matching function, and there is free entry into marketmaking.

Let α^M denote the measure of marketmakers, and then let $\overline{\mu}(\alpha^i, \alpha^M)$ denote the matching function which describes matches between market makers and searching buyers or searching sellers (where $\alpha^i \in \{\alpha^S, \alpha^B\}$ can describe the measure of sellers or buyers respectively). We define $\chi_S = \frac{\alpha^M}{\alpha^S}, \chi_B = \frac{\alpha^M}{\alpha^B}$ as the ratios of marketmakers to sellers and buyers, respectively, and use $\overline{\mu}_S(\chi_S)$ ($\overline{\mu}_M(\chi_S)$) to denote the rate at which a seller (marketmaker) is matched with a marketmaker (seller) and, analogously, use $\overline{\mu}_B(\chi_B)$ ($\overline{\mu}_M(\chi_B)$) to denote the rate at which a buyer (marketmaker) is matched with a marketmaker (buyer). We assume that $\overline{\mu}(\cdot,\cdot)$ satisfies the same properties as $\mu(\cdot,\cdot)$, such that the matching rates $\overline{\mu}_S(\cdot), \overline{\mu}_B(\cdot)$ satisfy the standard congestion properties (cf. equation (1) in the paper). Note that once we know the ratio of buyers to sellers and the ratio of marketmakers to buyers, we can compute the ratio of marketmakers to sellers ($\chi_S = \phi\chi_B$), such that the variables ϕ and χ_B will be sufficient to determine all matching intensities. The arrival of matches is independent and thus in a short time interval dt a marketmaker cannot be matched with both a seller and a buyer.

Market makers have discount rate ρ. There is free entry into marketmaking. While marketmakers search in the secondary market, they incur a flow utility cost of $e_M > 0$.

After a match involving a marketmaker, there is Nash bargaining, where the bargaining power parameter of marketmakers is γ, for both matches with buyers and matches with sellers. Taking into account the bargaining powers of the different agents and the equilibrium surpluses to be shared after a match, we can write the following equations that the prices must satisfy, as a function of the values that investors attach to holding the assets:

$$P - V_\rho = \beta(V_0 - V_\rho), \tag{A.1}$$

$$Q - B = \gamma(Q - V_\rho), \tag{A.2}$$

$$A - Q = \gamma(V_0 - Q). \tag{A.3}$$

These three equations together with the condition that the inter-dealer market must clear will determine the four prices A, B, Q, P.

In order for the inter-dealer market to clear, the equilibrium inter-dealer price Q must lie in the interval $[V_\rho, V_0]$.[1] To describe where in the relevant interval Q is located, it will be useful to define a variable λ that we call the *inter-dealer price index*, as follows:

$$\lambda := \lambda \in [0, 1] \text{ such that } Q = \lambda V_0 + (1 - \lambda)V_\rho.$$

To consider inter-dealer market clearing and the determination of λ (or equivalently Q) in more detail, it will be useful to sequentially consider the cases where $\phi < 1, \phi > 1$ or $\phi = 1$.

If $\phi < 1$, there are more sellers than buyers and thus there are more matches between marketmakers and sellers than between marketmakers and buyers. For the inter-dealer

[1] If Q lies outside this range, one type of match produces a negative surplus and hence no trade, while the other match produces a positive surplus, and hence trade. Hence marketmakers are only buying from or only selling to investors, which means that the inter-dealer market is not clearing.

market to clear, it must be the case that some of the matches between marketmakers and sellers will not lead to trade, which can happen only if there are no gains from trade associated with these matches, i.e. $\lambda = 0$ and $Q = V_\rho$.

If $\phi > 1$, there are more buyers than sellers and thus there are more matches between marketmakers and buyers than between marketmakers and sellers. For the inter-dealer market to clear, it must be the case that some of the matches between marketmakers and buyers will not lead to trade, which can happen only if there are no gains from trade associated to these matches, i.e. $\lambda = 1$ and $Q = V_0$.

If $\phi = 1$, there are as many buyers as sellers and thus there are as many matches between marketmakers and buyers as between marketmakers and sellers. In this case inter-dealer market clearing on its own is insufficient to pin down λ (or equivalently Q), but the fact that ϕ needs to be equal to 1 provides the necessary additional condition.

In all three cases, the system of recursive flow-value equations that V_0, V_ρ satisfy in steady state is as follows:

$$r + \delta(1 - V_0) + \theta(V_\rho - V_0) = 0 \tag{A.4}$$

$$r + \delta(1 - V_\rho) + \mu_S(\phi)(P - V_\rho) + \overline{\mu}_S(\chi_S)(B - V_\rho) = \rho V_\rho \tag{A.5}$$

The first equation is the same as equation (3) in the paper. The second equation corresponds to equation (4) in the paper, with the new term $\overline{\mu}_S(\chi_S)(B - V_\rho)$ that accounts for the possibility of locating a marketmaker and selling to him at the bid price B.

Using the conditions on the equilibrium prices P and B, the fact that free entry into the primary market auctions implies $V_0(r, \delta; \phi, \chi_B, \lambda) = 1$ (cf. equation (5) in the paper), and that $\chi_S = \phi\chi_B$, we can work out the interest rate as in Section 4.1, and obtain the following result (which is the analogue to Lemma 1 in the paper):

Lemma A.1. *In the model with marketmakers, for a given ratio of buyers to sellers ϕ, a given ratio of marketmakers to buyers χ_B, inter-dealer price index λ, and refinancing frequency choice δ of a firm, the interest rate $r(\delta; \phi, \chi_B, \lambda^e)$ that is set in the primary market* *auctions is given by:*

$$r(\delta;\phi,\chi_B,\lambda)=\frac{\rho}{\delta+\theta+\rho+\mu_S(\phi)\beta+\overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda}\theta. \tag{A.6}$$

As in the baseline model, the interest rate that a firm faces is determined by the bidding of investors in primary market auctions. The closed-form expression that can be derived is similar, but does not only depend on the ratio of buyers to sellers ϕ and the refinancing frequency δ, but also on the ratio χ_S of marketmakers to sellers. This is because impatient debt holders who search to sell can now be matched with a marketmaker, and sell at bid price B.

We can then follow the same procedure as in the baseline model to argue that firms exhaust their debt capacity and work out the optimal refinancing frequency choice, obtaining the following result (which is the analogue to Lemma 2):

Lemma A.2. *In the model with marketmakers, it is optimal to undertake the project and to issue debt. In addition, for every ϕ, χ_B, λ, the firm's problem analogous to* (13) *(in the paper) has a unique solution $\delta^*(\phi,\chi_B,\lambda)$ which is given by:*

$$\delta^*(\phi)=\max\left(\frac{1}{2}\left(\frac{x}{\kappa}-\theta-\rho-\mu_S(\phi)\beta-\overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda\right),0\right) \tag{A.7}$$

Similarly, a free entry condition for buyers corresponding to that in Lemma 4 can be derived:

Lemma A.4. *Free entry into the secondary market implies the following* free entry condition *for buyers:*

$$e_B=\left[\mu_B(\phi)(1-\beta)+\overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)\right]\frac{r(\delta;\phi,\chi_B,\lambda)}{\theta}. \tag{A.8}$$

Finally, a free entry condition for marketmakers can be derived, as in the following lemma:

Lemma A.5. *Free entry into marketmaking implies the following* free entry condition *for marketmakers:*

$$e_M=\left(\lambda\overline{\mu}_M(\phi\chi_B)+(1-\lambda)\overline{\mu}_M(\chi_B)\right)\gamma\frac{r(\delta;\phi,\chi_B,\lambda)}{\theta}. \tag{A.9}$$

A symmetric steady-state equilibrium of the economy in the model with marketmakers can be described by a tuple $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$ of refinancing frequency choice of firms δ^e, and ratios of buyers to sellers and marketmakers to buyers (ϕ^e, χ_B^e) together with an equilibrium inter-dealer price index $\lambda^e \in [0,1]$, such that the refinancing frequency choice of firms is optimal, the entry decisions of buyers and marketmakers are optimal, and the inter-dealer market clears (which amounts to $\lambda^e = 1$ if $\phi^e > 1$ and $\lambda^e = 0$ if $\phi^e < 1$).

In the model with marketmakers, a symmetric steady-state equilibrium exists, is unique, and is also inefficient, as summarized in the following two propositions (the analogues to Propositions 1 and 2 in the paper).

Proposition A.1. *In the model with marketmakers, there exists a unique symmetric steady-state equilibrium in the economy.*

Proposition A.2. *In the model with marketmakers, let $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$ describe a symmetric steady-state equilibrium with $\delta^e > 0$. Then the solution δ^{SP} to the Social Planner's problem satisfies $\delta^{SP} < \delta^e$.*

Finally, for any given equilibrium, the bid-ask spread $A - B$ of marketmakers can be calculated, as stated in the following lemma:

Lemma A.6. *In the model with marketmakers, the bid-ask spread is given by*

$$A - B = \gamma(V_0 - V_\rho) = \frac{\gamma\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta + \overline{\mu}_S(\phi\chi_B)(1-\gamma)\lambda}. \tag{A.10}$$

To interpret this bid-ask spread, consider e.g. the case in which $\phi > 1$, which implies that there are more buyers than sellers, and marketmakers consequently meet more buyers than sellers. This implies that marketmakers will set the ask price A equal to the reservation value of buyers, V_0, and the interdealer price Q will be equal to the ask price (and $\lambda = 1$), such that the expression becomes

$$A - B = \gamma(V_0 - V_\rho) = \frac{\gamma\rho}{\delta + \theta + \rho + \mu_S(\phi)\beta + \overline{\mu}_S(\chi_S)(1-\gamma)}. \tag{A.11}$$

As can be seen, the bid-ask spread is decreasing in the refinancing frequency δ, and hence increasing in maturity — this implication is consistent with Edwards, Harris, and Piwowar's (2007) evidence on estimated corporate bond transaction price spreads. (Also, the bid-ask spread is a constant fraction of the gains from trade, increasing in the bargaining power of marketmakers, and decreasing in the ratio of marketmakers to sellers.)

We now illustrate the analytical results via a numerical example of the extended the model with marketmakers. We hold fixed the parameter values and matching function between buyers and sellers used in the numerical example of the baseline model: $x = 1\%, \rho = 10\%, \theta = 1, \kappa = 3\text{bp}, \beta = 1/2, e_B = 5\%$ and $\mu(\alpha^B, \alpha^S) = 10\left(\alpha^B\right)^{1/2}\left(\alpha^S\right)^{1/2}$.

To these baseline parameterization we add the following matching function for marketmakers:

$$\overline{\mu}(\alpha^i, \alpha^M) = 100\left(\alpha^i\right)^{\frac{1}{2}}\left(\alpha^M\right)^{\frac{1}{2}},$$

where $i = \{B, S\}$. We note that this makes the matching technology of marketmakers 10 times more efficient, in the sense that for the same measures, marketmakers would obtain 10 times the match rate. We also assume that their flow cost of searching is much higher than that of buying investors and set $e_M = 50\%$, and that they have high bargaining power, and set $\gamma = 0.95$. The high cost of searching could be interpreted as justifying both the access to the more efficient matching technology and the high bargaining power after a match as well as the access to the inter-dealer market.

With these parameters, we have an equilibrium refinancing frequency of $\delta^{e,MM} \approx 11.16$ (vs $\delta^e \approx 13.04$ in the baseline model). This implies an expected maturity of debt claims of about $1/\delta^{e,MM} \approx 33$ days (vs $1/\delta^e \approx$28 days). The ratio of buyers to sellers is $\phi^{e,MM} \approx 1.27$, implying more sellers than buyers. The ratio of marketmakers to sellers is $\chi_S^{e,MM} \approx 0.73$. These ratios imply that the expected time for a selling investor to contact and trade with any counterparty (buying investor or marketmaker) is 3.8 days (vs about 30 days in the baseline model), and that a fraction of about 88% of all sales are to marketmakers and only 12% are direct to buyers.[2] The interest rate / illiquidity premium r that firms have to pay at this maturity of 33 days is equal to about 45bp (vs 49bp at the shorter maturity of 28 days in the baseline model). Entrepreneur utility is equal to U_{MM} ≈0.475 (vs $U \approx 0.235$). Compared to the baseline model, we observe that the presence of marketmakers increases the speed of trading in the secondary market. This allows firms to pay a lower interest rate even though they lengthen the expected maturity of debt, so that entrepreneur utility is increased.

To understand prices, first note that in this example, the value that an impatient investor attaches to the security is V_ρ ≈0.9955. The value that a patient investor attaches to the security is V_0=1, so that the gains from trade are about 45bp. The price that investors who meet directly agree to when they trade is equal to $P \approx 0.9977$, indicating that because of the equal bargaining power, they equally divide the gains from trade (slightly more than 22bp per party).

Because there are more buyers than sellers, marketmakers will find more buyers than sellers. As a response to the relative abundance of sellers, marketmakers will set the ask price A to the reservation value $V_0 = 1$ of buyers. The relative abundance of buyers also implies that the inter-dealer market price Q will equal the ask price $A = V_0 = 1$. Due to their high bargaining power, marketmakers will set a low bid price of $B \approx 0.9957$, leaving a seller who meets a marketmaker with a very small gain from the resulting trade of only 2bp, and keeping the bid-ask spread of 43bp as an intermediation profit.

In this situation, a social planner would choose a refinancing frequency of $\delta^{SP} \approx 3.34$, implying an expected maturity of around 109 days. This would induce entry of buyers and of marketmakers, leading to a ratio of buyers to sellers of $\phi^{SP} = 2.11$, and a ratio of marketmakers to sellers of χ_S^{SP} =1.22. The expected time for a selling investor to contact any counterparty is 2.9 days. In comparison to the laissez-faire equilibrium, the time to trade is decreased.

The longer maturity increases the gains from trade from about 45bp to about 58bp. The

[2] For buying investors, the expected time until they contact and trade with any counterparty (selling investor or marketmaker) is 4.3 days.

bid and ask prices are B ≈0.9945 and A =1, implying a larger bid-ask spread of 55bp, which sustains the increased numbers of marketmakers. The inter-dealer market price is still $Q = 1$. (The price at which investors trade is P ≈0.9971)

The interest rate / illiquidity premium r that firms have to pay at this new maturity of 109 days is equal to 58bp. At the same time, an individual firm that considers deviating from the laissez-faire equilibrium would perceive the interest rate required for issuing debt at a maturity of 109 days to be 70bp. The difference (70bp versus 58bp) arises because a coordinated increase in maturity choice increases gains from trade in the secondary market and encourages the entry of buyers and marketmakers into this market.

Finally, even though the interest rate at the socially regulated maturity is higher than in the laissez-faire economy, entrepreneurs benefit because the longer maturity allows them to save on the refinancing cost, leading to a higher entrepreneur utility of U ≈0.549. This represents an increase in entrepreneur utility of around 16% over the laissez-faire equilibrium.

B Increasing-returns-to-scale matching function

We start by describing how equilibrium can be determined for a general matching function in our baseline model. We then look at the increasing-returns-to-scale matching function $\mu^{IRS}(\alpha^S, \alpha^B) = \lambda\alpha^S\alpha^B$ for some $\lambda > 0$, which is the matching function used e.g. by Duffie, Garleanu, and Pedersen (2005). We will refer to this matching function as the increasing-returns-to-scale (IRS) matching function, and the matching function in the baseline model as the constant-returns-to-scale (CRS) matching function.

B.1 Equilibrium determination for general matching function

We first describe how the equilibrium is characterized for a general matching function $\mu(\alpha^S, \alpha^B)$. We let $\mu_S(\alpha^S, \alpha^B) := \frac{\mu(\alpha^S,\alpha^B)}{\alpha^S}$, $\mu_B(\alpha^S, \alpha^B) := \frac{\mu(\alpha^S,\alpha^B)}{\alpha^B}$ denote the marginal matching rates for a seller and a buyer, respectively, as in the main paper. For the sake of compactness we may sometimes only write μ_S, μ_B without explicit reference to the arguments of these function.

The characterization of equilibrium is similar to the one for the baseline model. For most of the equations relevant for the equilibrium determination it suffices to substitute the expressions for the marginal matching intensities in the CRS case with those in the general case. The most substantive difference is that with a general matching function the law of motion of the measure of sellers becomes relevant for characterizing equilibrium.

Interest rate function For a given tuple $(\delta, \alpha^S, \alpha^B)$ the expression for the interest rate is

$$r(\delta, \alpha^S, \alpha^B) = \frac{\rho}{\delta + \theta + \rho + \beta\mu_S(\alpha^S, \alpha^B)}\theta. \tag{B.1}$$

Optimal face value For a given tuple $(\delta, \alpha^S, \alpha^B)$ with $\delta \leq \frac{x}{\kappa}$ firms find it optimal to exhaust their debt capacity and choose a face value of debt

$$D(\delta, \alpha^S, \alpha^B) = \frac{x - \delta\kappa}{r(\delta, \alpha^S, \alpha^B)} \tag{B.2}$$

Optimal refinancing frequency For a given pair (α^S, α^B) the expression for the optimal refinancing frequency is

$$\delta^*(\alpha^S, \alpha^B) = \max\left\{\frac{1}{2}\left(\frac{x}{\kappa} - \theta - \rho - \beta\mu_S(\alpha^S, \alpha^B)\right), 0\right\}. \tag{B.3}$$

Free entry of buyers into the secondary market Free entry of buyers into the secondary market imposes the following restriction on the equilibrium tuples $(\delta, \alpha^S, \alpha^B)$:

$$e \geq \mu_B(\alpha^S, \alpha^B)(1-\beta)\frac{r(\delta, \alpha^S, \alpha^B)}{\theta} \text{ with equality if } \alpha^B > 0. \tag{B.4}$$

The inequality (which contrasts with the equality in the CRS case) accounts for the possibility of no entry of buyers ($\alpha^B = 0$) when the expected gains from becoming an active buyer do not compensate for the searching costs. This may happen in equilibrium because, as opposed to the CRS case, when $\alpha^B \to 0$ the buyers' matching intensity does not necessarily tend to infinity.

Stationarity of measure of sellers in equilibrium Conditions (B.2)-(B.4) are not sufficient to characterize the steady state equilibrium in the IRS case. An additional condition is required that comes from imposing stationarity of the measure of sellers. To find the expression for this condition we can write the law of motion of the measure of sellers:

$$\left(\alpha^S\right)' = \theta(D - \alpha^S) - \delta\alpha^S - \mu(\alpha^S, \alpha^B).$$

The first term on the right hand side accounts for the patient debtholders that were not active sellers and become impatient and hence active sellers, the second for the sellers that obtain liquidity through maturity and the last one for the sellers that find a buyer and sell their debt contracts. In a steady state equilibrium we have $\left(\alpha^S\right)' = 0$ and hence

$$\theta\left(D - \alpha^S\right) = \delta\alpha^S + \mu(\alpha^S, \alpha^B). \tag{B.5}$$

Equilibrium A laissez-faire symmetric steady-state equilibrium of the economy is characterized by a tuple $(\delta^e, D^e, \alpha^{S,e}, \alpha^{B,e})$ satisfying (B.2)-(B.5) with $\alpha^{S,e} \in [0, D^e], \alpha^{B,e} \geq 0$.

B.2 The economy with the IRS matching function

We will now use the IRS matching function $\mu^{IRS}(\alpha^S, \alpha^B) = \lambda\alpha^S\alpha^B$ for some $\lambda > 0$. With this matching function, we have that:

$$\mu_S^{IRS}(\alpha^B) = \lambda\alpha^B \text{ and } \mu_B^{IRS}(\alpha^S) = \lambda\alpha^S.$$

As in the CRS case, the equilibrium of the economy can be characterized as the intersection of two curves in a two-dimensional space. To obtain such simple equilibrium characterization we need the following lemma (analogous to Lemma 4 in the paper, albeit more involved from a technical perspective):

Lemma B.7. *For every $\delta \in \left[0, \frac{x}{\kappa}\right]$ the equilibrium conditions (B.2), (B.4) and (B.5) with $\alpha^S \in [0, D], \alpha^B \geq 0$ have a unique solution that defines a tuple of functions $\widehat{D}(\delta), \widehat{\alpha}^S(\delta), \widehat{\alpha}^B(\delta)$. Moreover, we have that*

$$\widehat{\alpha}^B(\delta) = \max\left(\frac{(x - \delta\kappa)(1 - \beta)}{e} - \frac{\theta + \delta}{\lambda}, 0\right)$$

and, in particular, if $\widehat{\alpha}^B(\delta) > 0$ *then* $\frac{d\widehat{\alpha}^B(\delta)}{d\delta} < 0$.

Using the lemma and the fact that the firms' optimal refinancing frequency only depends on α^B, we can characterize an equilibrium of the economy by a pair $(\delta^e, \alpha^{B,e})$ satisfying

$$\delta^e = \delta^*(\alpha^{B,e}) \text{ and } \alpha^{B,e} = \widehat{\alpha}_B(\delta^e).$$

We have:

Proposition B.3. *If* $\frac{e}{\lambda\kappa(1-\beta)+e} > \frac{1}{2}\beta$ *the equilibrium of the laissez-faire economy with IRS matching function exists and is unique.*

There are two dimensions in which the debt structure decision (δ, D) of firms exhibits strategic complementarities. As in the baseline model, if firms expect α^B to be low (high) they find it optimal to choose a high (low) δ, and there are small (large) gains from trade in the secondary market. But a high (low) δ implies a lot of (very little) entry by buyers into the secondary market, and hence a low (high) α^B. An additional source of strategic complementarities (not present in the CRS case) is that if firms expect α^B to be high (low), then they anticipate that interest rates will be low (high) and hence choose a large (small) face values of debt. Now, if firms choose a large (small) face value of debt, then by itself this will produce a high (low) measure of sellers α^S in the secondary market. Due to the IRS matching technology, this increases (decreases) the matching rate of buyers in the secondary market, such that even more (even less) buyers enter and hence also tends to produce a high (low) α^B. We are able to prove that despite the two layers of strategic complementarities the equilibrium is unique if the condition in the proposition is satisfied.[3]

B.3 Inefficiency of equilibrium

The efficiency analysis can be conducted as in the case with the CRS matching function. A social planner (SP) chooses a debt structure $\{(\delta, D)\}$ in order to maximize aggregate surplus,

[3] When the condition is not satisfied there exist generically either one or three equilibria. Moreover, when there are three equilibria, one of them has $\delta^e > 0, \alpha_B^e = 0$, another one has $\delta^e = 0, \alpha_B^e > 0$ and the third one has $\delta^e > 0, \alpha_B^e > 0$ and is unstable.

which coincides with firms' profits. The key difference with respect to the choice by private firms is that that the SP takes into account the effects of the debt structure decision on the equilibrium measure of buyers in the secondary market.

It can be shown that for every refinancing frequency choice δ, the SP finds it optimal to exhaust debt capacity.[4] Also, using Lemma B.7, we know that $\frac{d\widehat{\alpha}_B(\delta)}{d\delta} < 0$ if $\alpha^B > 0$, that is, increasing the refinancing frequency will decrease the measure of buyers. Furthermore, the interest rate $r(\delta, \alpha^B)$ is decreasing in the measure of buyers α^B. Together, these lead to the following inefficiency result:

Proposition B.4. *Suppose* $\frac{e_B}{\lambda\kappa(1-\beta)+e} > \frac{1}{2}\beta$ *and let* $(\delta^e, D^e, \alpha^{S,e}, \alpha^{B,e})$ *be an equilibrium with* $\delta^e > 0$ *and* $\alpha^{B,e} > 0$*. Then the solution* (δ^{SP}, D^{SP}) *to the Social Planner's problem satisfies* $\delta^{SP} < \delta^e$ *and* $D^{SP} > D^e$*.*

B.4 Comparison of quantitative magnitude of inefficiency

We now compare the quantitative magnitude of the inefficiencies in an IRS economy with those in our baseline CRS economy. In order to do so we would like to construct an IRS economy whose equilibrium coincides with the one in the numerical illustration of the CRS economy. The following result will help with setting up a meaningful comparison:

Lemma B.8. *Let* μ *be a matching function and* $(\delta^e, D^e, \alpha^{S,e}, \alpha^{B,e})$ *an equilibrium of this economy with* $\alpha^{B,e} > 0$*. Let* μ' *be another matching function. If* $\mu'_S(\alpha^{S,e}, \alpha^{B,e}) = \mu_S(\alpha^{S,e}, \alpha^{B,e})$ *then* $(\delta^e, D^e, \alpha^{S,e}, \alpha^{B,e})$ *is also an equilibrium of the economy with matching function* μ'*.*

This says the following: Suppose you have constructed an equilibrium for a given matching function. Now choose parameters for a new matching function, so that at the (old) equilibrium measures of buyers and sellers, the matching rate for sellers is the same for both matching functions. Then the equilibrium that you compute with the new matching function will be the same as the equilibrium that you computed from the original matching function.

[4] The argument is more involved that in the baseline model because with an IRS matching function, the face value of debt D issued by firms has an effect on the equilibrium measure of buyers, and hence on the interest rate faced by firms. See the proof of Proposition B.4 for details.

We use this to compare the IRS case with the CRS case in the numerical example in our paper. We chose the following parameter values and matching function: : $x = 1\%, \rho = 10\%, \theta = 1, \kappa = 3\text{bp}, \beta = 1/2, e_B = 5\%$ and $\mu^{CRS}(\alpha^B, \alpha^S) = 10\left(\alpha^B\right)^{1/2}\left(\alpha^S\right)^{1/2}$. The associated tuple of endogenous equilibrium variables was: $\delta^e_{CRS} \approx 13.04, D^e_{CRS} \approx 1.24, \alpha^{S,e}_{CRS} \approx 0.047, \alpha^{B,e}_{CRS} \approx 0.071$. From the lemma above we have that in order to construct an IRS economy with equilibrium tuple $(\delta^e_{CRS}, D^e_{CRS}, \alpha^{S,e}_{CRS}, \alpha^{B,e}_{CRS})$ it suffices to ensure that

$$\begin{aligned} \mu_S^{IRS}(\alpha^{S,e}_{CRS}, \alpha^{B,e}_{CRS}) &= \mu_S^{CRS}(\alpha^{S,e}_{CRS}, \alpha^{B,e}_{CRS}) \Leftrightarrow \\ \lambda \alpha^{B,e}_{CRS} &= 10\left(\frac{\alpha^{B,e}_{CRS}}{\alpha^{S,e}_{CRS}}\right)^{1/2} \approx 12.32, \end{aligned}$$

which requires

$$\lambda \approx 173.13. \tag{B.6}$$

Moreover, since the exogenous parameters satisfy the conditions that ensure uniqueness of equilibrium for the IRS economy in Proposition B.3 we conclude that $(\delta^e_{CRS}, D^e_{CRS}, \alpha^{S,e}_{CRS}, \alpha^{B,e}_{CRS})$ is the unique equilibrium of the IRS economy with multiplicative parameter λ given by (B.6).

In this economy, we find that a SP would choose a refinancing frequency $\delta^{SP}_{IRS} = 0$, i.e. it would find it optimal to force firms to issue perpetual debt (while the laissez-faire equilibrium has an expected maturity of 28 days). In the corresponding CRS economy the SP chose a refinancing frequency $\delta^{SP}_{CRS} \approx 8.90$, which implied an expected maturity of 41 days. So in the IRS economy, the SP finds it optimal to choose a much longer maturity. This is because of the thick market externalities/ lack of congestion effects in the IRS matching function: As the SP increases debt maturity, the increase in gains from trade attracts more buyers, which reduces interest rates and allows firms to issue more debt. Since with the IRS matching function, entering buyers to not impose congestion on buyers that are already in the secondary market, more of them can enter than in the CRS case, and the quantitative magnitude of the inefficiency increases.

The entrepreneur's utility increases from a laissez-faire level of $U \approx 0.236$ to a utility when all firms issue perpetual debt of $U^{SP}_{IRS} \approx 1.223$, which constitutes an increase of 420%. In contrast, in the CRS economy the utility of the entrepreneur under the regulated (expected) maturity of 41 days is $U_{CRS}^{SP} \approx 0.264$, which constitutes an increase of only 12% with respect to the laissez-faire case.

C Proofs

C.1 Marketmakers

Proof of Lemma A.1. Solve (A.4) and (A.5) for V_0, then find the r that satisfies $V_0(r, \delta; \phi, \chi_B, \lambda) = 1$ to obtain the stated result. ■

Proof of Lemma A.2. In the model with marketmakers, the interest rate is equal to or lower than the interest rate in the model without market makers. Hence it follows from the fact that under Assumption 1 in the paper, undertaking debt-financed projects in the model without marketmakers is optimal (Lemma 2 in the paper) that it is also optimal to do so here. Finally, with some simple algebra paralleling that in the proof of Lemma 2 in the paper, the stated expression for $\delta^*(\phi, \chi_B, \lambda)$ can be obtained. ■

Proof of Lemma A.4. Following the same procedure as for Lemma 4 in the paper, we note that $V_B = 0$ and hence that

$$\begin{aligned} e_B &= \mu_B(\phi)(V_0 - P) + \overline{\mu}_B(\chi_B)(V_0 - A) \\ &= \mu_B(\phi)(1-\beta)(V_0 - V_\rho) + \overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)(V_0 - V_\rho) \\ &= \left[\mu_B(\phi)(1-\beta) + \overline{\mu}_B(\chi_B)(1-\gamma)(1-\lambda)\right] \frac{r(\delta; \phi, \chi_B, \lambda)}{\theta}. \end{aligned} \tag{C.1}$$

In the first equation we now take into account that buyers can now also be matched with marketmakers, in the second, we use the conditions on P and A, and in the third equation we use the fact that $V_0 = 1$ as well as the definition of r. ■

Proof of Lemma A.5. The utility flow to a marketmaker is

$$\overline{\mu}_M(\chi_S)(Q - B) + \overline{\mu}_M(\chi_B)(A - Q) - e_M,$$

reflecting that marketmakers can be matched either with sellers and buyers, and incur the (non-pecuniary) flow cost e_M.

Free entry will occur until in equilibrium this utility flow is zero, implying

$$\begin{aligned} e_M &= \overline{\mu}_M(\chi_S)(Q-B) + \overline{\mu}_M(\chi_B)(A-Q) = \\ e_M &= \overline{\mu}_M(\phi\chi_B)\lambda\gamma(V_0 - V_\rho) + \overline{\mu}_M(\chi_B)\gamma(1-\lambda)(V_0 - V_\rho) = \\ &= (\lambda\overline{\mu}_M(\phi\chi_B) + (1-\lambda)\overline{\mu}_M(\chi_B))\,\gamma\frac{r(\delta;\phi,\chi_B,\lambda)}{\theta}, \end{aligned} \tag{C.2}$$

where we have used the conditions on prices and the fact that $\chi_S = \phi\chi_B$, and that $V_0 = 1$, as well as the relationship between V_0, V_ρ and r. ■

Proof of Proposition A.1. The liquidity conditions in the secondary market are described by the variables ϕ, χ_B and λ. The idea behind the proof is to reduce this set of variables to a single variable y, and then, as in the baseline model, to characterize equilibria as the intersection in the (y,δ)-space of a curve describing the solution to the firm's problem, and a curve describing free entry.

In order to construct the variable y we have first to make some definitions and manipulate equations (A.7), (C.1), and (A.9).

We define

$$\chi := \begin{cases} \chi_B \text{ if } \phi \leq 1 \\ \chi_S \text{ if } \phi > 1 \end{cases}.$$

Using the variable χ, and the fact that $\phi < 1 \implies \lambda = 0$, $\phi = 1 \implies \lambda \in (0,1)$, and $\phi > 1 \implies \lambda = 1$, we can re-write the free entry condition for marketmakers (A.9) as

$$e_M = \overline{\mu}_M(\chi)\gamma\frac{r(\delta;\phi,\chi_B,\lambda)}{\theta}, \tag{C.3}$$

as well as the free entry condition for buyers. Dividing the free entry condition for buyers by that for marketmakers, we now obtain:

$$\frac{e_B}{e_M} = \frac{\mu_B(\phi)(1-\beta) + \overline{\mu}_B(\chi)(1-\gamma)(1-\lambda)}{\overline{\mu}_M(\chi)\,\gamma}. \tag{C.4}$$

It is easy to see that for every ϕ, λ there exists a unique χ such that the equation above is satisfied. Denote the function this equation defines by $\chi^{FEC}(\phi,\lambda)$. $\chi^{FEC}(\phi,\lambda)$ is continuous for $\phi \in (0,1)$ and $\phi \in (1,\infty)$ and for $\lambda \in [0,1]$, it is increasing in ϕ and is also increasing in λ. In addition:

$$\begin{aligned}
\lim_{\phi \to 1^-} \chi^{FEC}(\phi,\lambda) &= \chi^{FEC}(1,0) = \lim_{\lambda \to 0^+} \chi^{FEC}(1,\lambda), \\
\lim_{\phi \to 1^+} \chi^{FEC}(\phi,\lambda) &= \chi^{FEC}(1,1) = \lim_{\lambda \to 1^-} \chi^{FEC}(1,\lambda), \\
\lim_{\phi \to 0^+} \chi^{FEC}(\phi,\lambda) &= 0, \\
\lim_{\phi \to \infty} \chi^{FEC}(\phi,\lambda) &= \infty.
\end{aligned}$$

We now define:

$$F(y) = \begin{cases} (y,0) \text{ if } y \in (0,1) \\ (1, y-1) \text{ if } y \in [1,2] \\ (y-1, 1) \text{ if } y > 2 \end{cases}$$

The function $\chi^{FEC}(F(y))$ is continuous and strictly increasing in $y \in (0,\infty)$. It also satisfies:

$$\begin{aligned}
\lim_{y \to 0^+} \chi^{FEC}(F(y)) &= 0, \\
\lim_{y \to \infty} \chi^{FEC}(F(y)) &= \infty.
\end{aligned}$$

Define the function:

$$G(y) = \begin{cases} y \text{ if } y \in (0,1) \\ 1 \text{ if } y \in [1,2] \\ y-1 \text{ if } y > 2 \end{cases}$$

$G(y)$ is a continuous function.

We can use these definitions to rewrite the optimal refinancing frequency choice in equation (A.7) as

$$\delta^*(y) = \max\left\{ \frac{1}{2}\left(\frac{x}{\kappa} - \theta - \rho - \mu_S(G(y))\beta - \left(\mathbf{1}_{\{y>2\}} + \mathbf{1}_{\{y\in[1,2]\}}(y-1)\right)\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma)\right), 0\right\},$$

and to rewrite the free entry condition for buyers in equation (C.1) as

$$\begin{aligned}
\delta^{FEC}(y) = {} & \frac{\rho}{e_B}\left[\mu_B(G(y))(1-\beta) + \left(\mathbf{1}_{\{y<1\}} + \mathbf{1}_{\{y\in[1,2]\}}(2-y)\right)\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma)\right] \qquad \text{(C.5)} \\
& -\theta - \rho - \mu_S(G(y))\beta - \left(\mathbf{1}_{\{y>2\}} + \mathbf{1}_{\{y\in[1,2]\}}(y-1)\right)\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma).
\end{aligned}$$

Both functions are continuous.

It is easy to convince oneself that the definitions of $G(y)$, $\chi^{FEC}(F(y))$, $\delta^*(y)$ and $\delta^{FEC}(y)$ are such that if (y^e, δ^e) satisfy:

$$\delta^e = \delta^*(y^e) = \delta^{FEC}(y^e), \tag{C.6}$$

then, if $y^e < 1$ the tuple $(\delta^e, \phi^e = G(y^e), \chi_B^e = \chi^{FEC}(F(y^e), \lambda^e = 0)$ is an equilibrium, if $y^e > 2$ the tuple $(\delta^e, \phi^e = G(y^e), \chi_B^e = G(y^e)\chi^{FEC}(F(y^e), \lambda^e = 1)$ is an equilibrium and finally if $y^e \in [1,2]$ the tuple $(\delta^e, \phi^e = 1, \chi_B^e = \chi^{FEC}(F(y^e), \lambda^e = y^e - 1)$ is an equilibrium. Conversely, for every equilibrium $(\delta^e, \phi^e, \chi_B^e, \lambda^e)$, we define $y^e = \phi^e$ if $\phi^e < 1$, define $y^e = 1 + \lambda^e$ if $\phi^e = 1$, and $y^e = \phi^e + 1$ if $\phi^e > 1$. Then (δ^e, y^e) satisfy equation (C.6).

Therefore, pairs (δ^e, y^e) satisfying equation (C.6) characterize the equilibria of the model. In order to prove the proposition it suffices to prove that the functions $\delta^*(y), \delta^{FEC}(y)$ have a unique intersection point in the interval $(0, \infty)$. From this point onwards the proof is analogous to the one of Proposition 1.

First, existence is consequence of the continuity of both functions and their behaviour at the limits of the interval $(0, \infty)$:

$$\begin{aligned} \lim_{y \to 0^+} \delta^*(y) &< \lim_{y \to 0^+} \delta^{FEC}(y) = \infty, \\ \lim_{y \to \infty} \delta^*(y) &= 0 > \lim_{y \to \infty} \delta^{FEC}(y) = -\infty. \end{aligned}$$

Second, in order to prove uniqueness it suffices to prove that the inequality

$$\frac{d\delta^{FEC}(y)}{dy} < \frac{d\delta^*(y)}{dy},$$

is satisfied almost everywhere.[5] Comparing the analytical expressions for $\delta^*(y)$ and $\delta^{FEC}(y)$ and taking into account that

$$\frac{d}{dy}\left(\mu_S(G(y))\beta + \left(\mathbf{1}_{\{y>2\}} + \mathbf{1}_{\{y\in[1,2]\}}(y-1)\right)\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma)\right) > 0,$$

it suffices to prove that

$$\frac{d}{dy}\left[\mu_B(G(y))(1-\beta) + \left(\mathbf{1}_{\{y<1\}} + \mathbf{1}_{\{y\in[1,2]\}}(2-y)\right)\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma)\right] < 0. \tag{C.7}$$

[5] The functions are not differentiable at $y = 1, 2$ and at the smallest y for which $\delta^*(y) = 0$.

Since $\mu_B(\cdot)$ is decreasing in its argument but $\overline{\mu}_B(\cdot)$ is increasing the sign of the expression above is ambiguous. However, we can rewrite equation (C.4) as:

$$\mu_B(G(y))(1-\beta)+\left(\mathbf{1}_{\{y<1\}}+\mathbf{1}_{\{y\in[1,2]\}}(2-y)\right)\overline{\mu}_B(\chi^{FEC}(F(y)))(1-\gamma)=\overline{\mu}_M\left(\chi^{FEC}(F(y))\right)\gamma\frac{e_B}{e_M}$$

and insert the result into the inequality. Since $\chi^{FEC}(F(y))$ is strictly increasing in y and $\overline{\mu}_M(\cdot)$ is strictly decreasing in its argument, it is clear that the inequality holds.

■

Proof of Proposition A.2. We work in the (y,δ)-space defined in the proof of the previous proposition. The equilibrium can be described by a pair (y^e,δ^e). The function $\delta^{FEC}(y)$ defined in equation (C.5) is strictly decreasing. Let $y^{FEC}(\delta)$ be its inverse function, which is defined for $\delta \geq 0$, strictly decreasing and differentiable except at $\delta = \delta^{FEC}(1)$ and $\delta = \delta^{FEC}(2)$. The social planner finds it optimal to exhaust debt capacity for any refinancing frequency choice (for the same reasons as individual firms do), and so maximizes:

$$U^{SP}(\delta)=U(\delta;y^{FEC}(\delta))=-1-\kappa+\frac{x-\delta\kappa}{r(\delta;y^{FEC}(\delta))},$$

where $r(\delta;y)$ has the following expression:

$$r(\delta;y)=\frac{\theta}{\delta+\theta+\rho+\mu_S(G(y))\beta+\left(\mathbf{1}_{\{y>2\}}+\mathbf{1}_{\{y\in[1,2]\}}(y-1)\right)\overline{\mu}_S(\chi^{FEC}(F(y)))(1-\gamma)}\rho.$$

Taking into account that $G(y)$ is increasing in y and that $\chi^{FEC}(F(y))$ is strictly increasing in y we have that:

$$\frac{\partial r(\delta;y)}{\partial y}<0\Leftrightarrow\frac{\partial U(\delta;y)}{\partial y}>0.$$

From here, we have as in the baseline model that if $\delta^e>0$ then:

$$\left.\frac{dU^{SP}(\delta)}{d\delta}\right|_{\delta=\delta^e}<0.$$

The same arguments as in the proof of Proposition 2 where $\phi,\phi^{FEC}(\delta)$ are replaced by $y,y^{FEC}(\delta)$ also lead to:

$$U^{SP}(\delta)<U^{SP}(\delta^e)\text{ for all }\delta>\delta^e.$$

We conclude from the two previous inequalities that:

$$\arg\max_{\delta\geq 0} U^{SP}(\delta) < \delta^e.$$

■

Proof of Lemma A.6. Calculate $A-B$ directly from (A.2) and (A.3), and use $\theta(V_0-V_\rho)=r$ and equation (A.6) to obtain the stated expression. ■

C.2 Increasing-returns-to-scale matching function

Proof of Lemma B.7. We first observe that with an IRS matching function the interest rate function only depends on (δ,α^B) and the optimal refinancing frequency only depends on α^B.

For a given δ, let us denote $\widehat{D},\widehat{\alpha}^S,\widehat{\alpha}^B$ a solution to the set of conditions in the lemma.

Suppose that $\widehat{\alpha}^S>0,\widehat{\alpha}^B>0$. Then, substituting the expression for $\widehat{D}$ in (B.2) into (B.4) and (B.5) we have

$$e\theta = \lambda(1-\beta)r(\delta;\widehat{\alpha}^B)\widehat{\alpha}^S, \tag{C.8}$$

$$\theta\left(\frac{x-\delta\kappa}{r(\delta;\widehat{\alpha}^B)}-\widehat{\alpha}^S\right) = \left(\delta+\lambda\widehat{\alpha}^B\right)\widehat{\alpha}^S, \tag{C.9}$$

and isolating $r(\delta;\widehat{\alpha}^B)$ from the first expression and substituting it into the second one we obtain

$$\theta\left(\frac{(x-\delta\kappa)\,\lambda(1-\beta)}{e\theta}-1\right)\widehat{\alpha}^S=\left(\delta+\lambda\widehat{\alpha}^B\right)\widehat{\alpha}^S.$$

Using that $\widehat{\alpha}^S>0$ we get

$$\widehat{\alpha}^B=\frac{(x-\delta\kappa)\,(1-\beta)}{e}-\frac{\theta+\delta}{\lambda}, \tag{C.10}$$

which expresses $\widehat{\alpha}^B$ as a function of δ. Now, the RHS in the equation above is positive iff $\delta<\overline{\delta}$ where $\overline{\delta}$ is the unique solution to the following equation

$$\theta\left(\frac{(x-\overline{\delta}\kappa)\,\lambda(1-\beta)}{e\theta}-1\right)=\overline{\delta}, \tag{C.11}$$

and satisfies trivially that $\overline{\delta} < \frac{x}{\kappa}$. From (C.8), the expression for $r(\delta; \widehat{\alpha}^B)$ in (B.1) and the expression for $\widehat{\alpha}^B$ as a function of δ in (C.10), we can obtain $\widehat{\alpha}^S$ as a function of δ. Moreover, (C.8) and (C.9) trivially imply that $\widehat{\alpha}^S \in (0, \widehat{D})$.

Summing up, we have proved that for $\delta < \overline{\delta}$ there exists a unique solution to the conditions in the lemma with $\widehat{\alpha}^S > 0, \widehat{\alpha}^B > 0$ and (C.10) provides an explicit analytical expression for $\widehat{\alpha}^B$.

To conclude the proof of the lemma in the case $\delta < \overline{\delta}$ it suffices to show that there are no solutions with $\widehat{\alpha}^S = 0$ or $\widehat{\alpha}^B = 0$:

If $\widehat{\alpha}^S = 0$ then (B.5) implies that $\widehat{D} = \widehat{\alpha}^S = 0$ which in turn implies that $\delta = \frac{x}{\kappa} > \overline{\delta}$.

Suppose on the other hand that $\widehat{\alpha}^B = 0$. Then (B.2) determines $\widehat{D}$ and from (B.5) we have that $\widehat{\alpha}^S = \frac{\theta \widehat{D}}{\theta + \delta}$. Taking this into account we have that (B.4) requires that

$$\begin{aligned} e\theta &\geq \lambda(1-\beta) r(\delta; 0) \frac{\theta \widehat{D}}{\theta + \delta} \\ &= \lambda(1-\beta) \frac{\theta\,(x - \delta\kappa)}{\theta + \delta}. \end{aligned}$$

The inequality can be rewritten as

$$\theta \left(\frac{(x - \delta\kappa)\,\lambda(1-\beta)}{e\theta} - 1 \right) \leq \delta,$$

but from the equation (C.11) defining $\overline{\delta}$ and the assumption that $\delta < \overline{\delta}$, we deduce that this inequality is not satisfied.

Now suppose that $\overline{\delta} \leq \delta < \frac{x}{\kappa}$. The arguments above show that a solution to the conditions in the lemma has to satisfy $\widehat{\alpha}^S > 0$ and $\widehat{\alpha}^B = 0$. Also, reproducing those arguments we can prove that the tuple $\widehat{D}, \widehat{\alpha}^S, \widehat{\alpha}^B = 0$ where $\widehat{D}$ is given by (B.2) and $\widehat{\alpha}^S$ is given by (B.5), also satisfies (B.4) and $\widehat{\alpha}^S \in (0, \widehat{D})$. It is hence a solution to the conditions of the lemma, and therefore proves existence. Uniqueness is straightforward to prove.

Finally, the case $\delta = \frac{x}{\kappa}$ leads straightforwardly to $(\widehat{D}, \widehat{\alpha}^S, \widehat{\alpha}^B) = (0, 0, 0)$. ■

Proof of Proposition B.3. For $\delta < \overline{\delta}$ we can invert the function $\widehat{\alpha}^B(\delta)$ to obtain (after some algebra):

$$\widehat{\delta}(\alpha^B) = \frac{\lambda x(1-\beta) - e\theta - e\lambda\alpha^B}{\lambda\kappa(1-\beta)+e} \text{ for } \alpha^B \in (0, \overline{\alpha}^B]$$

with $\overline{\alpha}^B = \widehat{\alpha}^B(0)$.

Now, the assumption $\frac{e}{\lambda\kappa(1-\beta)+e} > \frac{1}{2}\beta$ ensures that the slope of the linear function $\widehat{\delta}(\alpha^B)$ is smaller (bigger in absolute value) than that of the piecewise linear function $\delta^*(\alpha^B)$, which ensures that there exists at most one equilibrium.

We now prove existence.

By construction we have $\overline{\delta} = \lim_{\alpha^B \to 0} \widehat{\delta}(\alpha^B)$. If $\overline{\delta} \leq \delta^*(0)$ then $\delta^e = \delta^*(0), \alpha^{B,e} = 0$ constitutes an equilibrium.

Suppose that $\overline{\delta} > \widehat{\delta}(0)$. We have by construction $\widehat{\delta}(\overline{\alpha}^B) = 0$. If $\delta^*(\overline{\alpha}^B) = 0$ then $\delta^e = 0, \alpha^{B,e} = \overline{\alpha}^B$ constitutes an equilibrium.

Otherwise, since by assumption $\lim_{\alpha^B \to 0} \widehat{\delta}(\alpha^B) = \overline{\delta} > \widehat{\delta}(0)$ and $\widehat{\delta}(\overline{\alpha}_B) = 0 < \delta^*(\overline{\alpha}^B)$, by continuity there exists $\alpha^{B,e} \in (0, \overline{\alpha}^B)$ such that $\widehat{\delta}(\alpha^{B,e}) = \delta^*(\alpha^{B,e})$, $\delta^e = \widehat{\delta}(\alpha^{B,e})$, and $\alpha^{B,e}$ constitutes an equilibrium. ■

Proof of Proposition B.4. Let (δ, D) be a debt structure chosen by the SP. By definition, the debt structure is feasible if and only if there exists a solution $\alpha^S \in [0, D], \alpha^B \geq 0$ to (B.4) and (B.5) such that $D \leq \frac{x-\delta\kappa}{r(\delta,\alpha^B)}$. (Notice that with an IRS matching function the interest rate only depends on α^B.) When this is the case, α^S, α^B are the steady state measures of sellers and buyers, respectively, induced by the debt structure (δ, D).

It is easy to prove that if a solution $\alpha^S \in [0, D], \alpha^B \geq 0$ to (B.4) and (B.5) exists, it is unique. For every feasible debt structure (δ, D) we can therefore define the functions $\widetilde{\alpha}^S(\delta, D), \widetilde{\alpha}^B(\delta, D)$ that determine the induced steady state measures of sellers and buyers.

We now prove that if (δ, D) is a socially optimal debt structure then the debt capacity of the firms is exhausted, i.e.

$$D = \frac{x - \delta\kappa}{r(\delta, \widetilde{\alpha}^B(\delta, D))}. \tag{C.12}$$

Suppose this is not the case. The entrepreneurs' profits are given by:

$$U(\delta, D) = -1 - \kappa + D + \frac{x - r(\delta, \widetilde{\alpha}^B(\delta, D))D - \delta\kappa}{\rho},$$

and we have that

$$\frac{\partial U(\delta, D)}{\partial D} = 1 - \frac{r(\delta, \widetilde{\alpha}^B(\delta, D))}{\rho} - \frac{D}{\rho}\frac{\partial r}{\partial \alpha^B}\frac{\partial \widetilde{\alpha}^B(\delta, D)}{\partial D}. \tag{C.13}$$

It can be easily checked out of the conditions (B.4) and (B.5) that $\frac{\partial \widetilde{\alpha}^B(\delta,D)}{\partial D} \geq 0$. Using this, and the inequalities $r(\delta, \widetilde{\alpha}^B(\delta, D)) < \rho$, $\frac{\partial r}{\partial \alpha^B} < 0$, we have from (C.13) that $\frac{\partial U(\delta,D)}{\partial D} > 0$. As a result, if the debt capacity of the firms is not exhausted an increase in face value of debt strictly increases the entrepreneurs' utility, which contradicts the assumption that (δ, D) is a socially optimal debt structure.

We can hence restrict ourselves to debt structures (δ, D) satisfying (C.12). By construction, we have then that $D = \widehat{D}(\delta)$, and that $\widetilde{\alpha}^B(\delta, D) = \widehat{\alpha}^B(\delta)$, where $\widehat{D}(\delta), \widehat{\alpha}^B(\delta)$ are defined in Lemma B.7. The SP problem hence becomes

$$\max_{\delta \geq 0} U^{SP}(\delta) = -1 - \kappa + \frac{x - \delta\kappa}{r(\delta, \widehat{\alpha}^B(\delta))}.$$

From here, the proof of the proposition is an immediate consequence of the arguments made in the text just before the proposition. ∎

Proof of Lemma B.8. The tuple $(\delta^e, D^e, \alpha^{S,e}, \alpha^{B,e})$ satisfies the equilibrium conditions (B.2)-(B.5) for the matching function μ. It suffices to show that $(\delta^e, D^e, \alpha^{S,e}, \alpha^{B,e})$ satisfies these conditions also for the matching function μ'. Taking into account that the interest rate function depends on the marginal matching rate of sellers (and not on that of buyers), that by assumption $\mu'_S(\alpha^{S,e}, \alpha^{B,e}) = \mu_S(\alpha^{S,e}, \alpha^{B,e})$, which in particular implies that $\mu'(\alpha^{S,e}, \alpha^{B,e}) = \mu(\alpha^{S,e}, \alpha^{B,e})$, we have that $(\delta^e, D^e, \alpha^{S,e}, \alpha^{B,e})$ trivially satisfies (B.2),(B.3) and (B.5) for the matching function μ'.

Taking into account that

$$\mu'_B(\alpha^{S,e}, \alpha^{B,e}) = \frac{\alpha^{S,e}}{\alpha^{B,e}}\mu'_S(\alpha^{S,e}, \alpha^{B,e}) = \frac{\alpha^{S,e}}{\alpha^{B,e}}\mu_S(\alpha^{S,e}, \alpha^{B,e}) = \mu_B(\alpha^{S,e}, \alpha^{B,e}),$$

and using the arguments above we have that $(\delta^e, D^e, \alpha^{S,e}, \alpha^{B,e})$ also satisfies (B.4) for the matching function μ'. ■

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