Consider the following constrained likelihood function

$$\mathcal{L} = \log L(\theta) + \lambda' G(\theta), \tag{1}$$

where G is the vector of active constraints, θ is a $P \times 1$ vector of parameters, and λ is a $R \times 1$ vector of Lagrange multipliers.

The first order condition is

$$\frac{\partial \log L(\hat{\theta})}{\partial \theta} - \hat{\lambda}' \dot{G}(\hat{\theta}) = 0 \tag{2}$$

$$G = 0 \tag{3}$$

Under some regularity conditions, $\hat{\theta} \to \theta_0$, and we can assume

$$\hat{\lambda}' \dot{G}(\hat{\theta}) \approx \hat{\lambda}' \dot{G}(\theta_0) \tag{4}$$

$$\frac{\partial \log L(\hat{\theta})}{\partial \theta} \approx \frac{\partial \log L(\theta_0)}{\partial \theta} + \frac{\partial^2 \log L(\hat{\theta})}{\partial \theta \partial \theta'} (\hat{\theta} - \theta_0) \tag{5}$$

$$G(\hat{\theta}) \approx \dot{G}(\theta_0)(\hat{\theta} - \theta_0) \tag{6}$$

Let

$$H = \frac{\partial^2 \log L}{\partial \theta \partial \theta'} \tag{7}$$

denote the Hessian of the (unconstrained) log likelihood, and

$$\dot{G} = \frac{\partial G}{\partial \theta}, \tag{8}$$

the Jacobian of the active constraints. Upon substituting our approximations and dividing by $\sqrt{n}$ and rearranging, we obtain

$$\begin{bmatrix} \sqrt{n}(\hat{\theta} - \theta_0) \\ \frac{1}{\sqrt{n}} \hat{\lambda} \end{bmatrix} \approx \begin{bmatrix} -\frac{1}{n} H & \dot{G}' \\ \dot{G} & 0 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\sqrt{n}} \frac{\partial \log L(\theta_0)}{\partial \theta} \\ 0 \end{bmatrix} \tag{9}$$

Since the score is normal with variance $-H$, our vector $(\theta, \lambda)'$ has variance

$$\begin{bmatrix} -\frac{1}{n} H & \dot{G} \\ \dot{G} & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\frac{1}{n} H & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{n} H & \dot{G} \\ \dot{G} & 0 \end{bmatrix}^{-1} \tag{10}$$

Let A denote the top left hand block ($P \times P$) of the inverse we need to calculate. Using the formula for the inverse of partitioned matrices yields

$$A = -n(H^{-1} - H^{-1} \dot{G}' \left(\dot{G} H^{-1} \dot{G}' \right) \dot{G} H^{-1}) \tag{11}$$

and

$$\text{Var}(\theta) = \frac{1}{n} A (-\frac{1}{n} H) A \tag{12}$$

defining $\bar{A} = -A/n$, $\text{Var}(\theta) = -\bar{A} H \bar{A}$, which after some simplification reduces to $\text{Var}(\theta) = \bar{A}$.